B.M.S COLLEGE FOR WOMEN AUTONOMOUS
BENGALURU – 560004

END SEMESTER EXAMINATION – OCTOBER 2022

M.Sc. in Mathematics – II Semester

Topology – II

Course Code: MM203T **QP Code: 21003**
Duration: 3 Hours **Max marks: 70**

Instructions: 1) All questions carry equal marks.
2) Answer any five full questions.

1. a) Define a compact space. Prove that a closed subset of a compact space is compact.
   b) Prove that every sequentially compact space is countably compact. Is the converse true? Explain.
   (7+7)
2. a) Prove that a metric space which is Lindelöf is second countable.
   b) Prove that second axiom space is both hereditary and topological property.
   (7+7)
3. a) Define projections on the product space $X \times Y$. Prove that product topology is the smallest topology for which the projections are continuous.
   b) Prove that if A is closed in (X, τ) and B is closed in (Y, τ^*) then $A \times B$ is closed in the product topology and conversely.
   (7+7)
4. a) Define T_0 −space. Prove that in a T_0 −space the closure of distinct points are distinct and conversely.
   b) Prove that a topological space is (X, τ) is a T_1 −space if and only if all singleton sets are closed.
   (7+7)
5. a) Prove that a continuous bijection from a compact space onto a Hausdorff space is a homeomorphism.
   b) Show that a metric space is a T_3 −space.
   (7+7)
6. a) Show that a regular Lindelöf space is normal.
   b) Show that a normal space is regular if and only if it is completely regular.
   (6+8)
7. a) Prove that complete normality implies normality.
   b) State and prove Tietze's extension theorem.
   (5+9)
8. a) Prove that a space is completely normal if and only if every subspace is normal.
   b) Prove that a regular Lindelöf space is paracompact.
   (7+7)

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